# Constraint-Induced Latent Competency Toward Euler's Number in Cellular Automata

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## Summary

Understanding the minimal conditions under which goal-directed behavior emerges is central to the study of competency. Here we examine an extremely simple cellular automaton in which cells follow a single local rule: each cell multiplicatively transfers its value to the largest neighboring cell. Despite the absence of any global controller or encoded objective, the system reliably converges toward a stable region near Euler's number $e$, as measured by a global reciprocal-sum metric. We show that locality is essential for stable convergence; extending interaction range beyond a cellular neighborhood induces cascade overshoot. Critically, introducing physical barriers improves convergence by up to 34%. Mechanistic analysis reveals that barriers enhance performance by limiting cascade depth and reducing participation, thereby constraining runaway amplification. These results demonstrate that appropriately scaled constraints can induce refined collective behavior in minimal systems, supporting the view that latent basal competencies can emerge from simple local interactions under structural constraint.

## Introduction

The field of Diverse Intelligence examines behavioral competencies across heterogeneous substrates, including neural systems, molecular and developmental processes, and computational or algorithmic collectives (Zhang et al., 2024; Levin, 2023a). A central problem in this literature concerns the minimal conditions under which goal-oriented convergence can emerge: how simple can a system be while still reliably moving toward a target state under perturbation?

Recent work by Zhang, Goldstein, and Levin (2024) demonstrated that classical sorting algorithms, when reinterpreted as distributed systems composed of locally acting agents, exhibit behavioral properties not explicit in their formal specifications. These include tolerance to perturbations, delayed gratification, and spontaneous clustering by behavioral type. A key insight from this work is that familiar algorithms can express novel competencies when instantiated as locally interacting collectives rather than as centralized procedures.

Critically, Zhang et al. showed that introducing constraints—specifically, "frozen cells" that are temporarily unable to move—revealed problem-solving behaviors that were absent in unconstrained executions. In response to these constraints, the system transiently moved away from its nominal goal to circumvent obstacles. Such detours are a canonical marker of genuine goal-directed behavior, long distinguished by William James (1890) from simple gradient-following dynamics.

This observation suggests a broader principle: constraint can expose or induce competencies that remain latent in unconstrained systems. In biological morphogenesis, for example, physical boundaries, tissue mechanics, and signaling constraints play an essential role in shaping robust anatomical outcomes (Levin, 2023a). Rather than merely impeding progress, obstacles can force a system to recruit additional degrees of freedom and alternative pathways, thereby revealing its underlying problem-solving capacity.

Here, we test whether this principle applies to an extremely minimal cellular automaton governed by a single local rule. Each cell inspects its neighborhood and multiplicatively transfers its value to the largest neighboring cell. We quantify global system behavior using a convergence metric, which is a modification of the infinite series of $e$,

$$S = 1 + \sum_i \frac{1}{v_i} \tag{1}$$

where $v_i$ is the value of cell $i$. $0!$ is replaced with $1$ and added to the reciprocal sum. It is worth noting the choice to work with integers at a cellular level–this series of experiments could have been conducted with floats and characterized as a simple sum of cells. Integers were chosen for simplicity in initialization and visualization.

We hypothesize that this system will exhibit emergent convergence toward a mathematically significant value, namely $e$, through collective behavior when adequetly constrained.

## Methods

### *Cellular Architecture and the Multiplication Rule*

We implement a 10×10 grid where each cell holds an integer value initialized randomly between 1 and 100. The system evolves through discrete rounds.

*The Multiplication Rule.* During each round, cells act in randomized order. An acting cell inspects neighbors within a Manhattan distance $d$ and identifies the neighbor with the largest value. The acting cell then multiplies that neighbor's value by its own:

$$v_{\text{target}} \times v_{\text{actor}} \rightarrow v_{\text{target}} \quad (2)$$

Importantly, all decisions are local: cells have no access to global state or objectives. This rule embodies a form of *local agency*: each cell makes decisions based solely on its immediate environment.

While the rule is purely mechanical, it instantiates a minimal form of agency, in which local interactions collectively shape global system behavior without centralized coordination.

### *Walls as Interaction Constraints*

Walls block perception and interaction between adjacent cells. A cell cannot inspect or modify neighbors across a wall. We tested six wall configurations: no walls (none), a horizontal wall (h_wall), a vertical wall (v_wall), a cross pattern (cross), double horizontal walls (rows 3 and 6, double_h), and a central enclosed 2×2 box (box).

Walls operationalize sub-areas by constraining the topology of possible interactions. Following Zhang et al. (2024), we hypothesize that appropriately scaled constraints will alter cascade dynamics and improve collective performance rather than degrade it. Thus, revealing an enhanced competency based on restriction.

### *Experimental Protocol*

Four experiments were conducted with 1,000 iterations each, ensuring statistical reliability (standard error $\approx 0.018$).

*Experiment 1:* Baseline trajectory analysis of the multiplication rule at $d = 1$.

*Experiment 2:* Inspection distance variation ($d = 1$ through $d = 5$).

*Experiment 3:* Wall configuration comparison using optimal distance.

*Experiment 4:* Propagation analysis tracking generation depth and participation rates to explain mechanistic differences.

## Results

### *Emergent Convergence Toward e*

Across runs in Experiment 1, the system consistently exhibited a decrease in $S$ toward values near Euler's number $e \approx 2.718$, achieving a mean final value of $S = 2.557$ (SD = 0.564), 0.161 from the mathematical constant (Figure 1).

Notably, the update rule contains no reference to $e$ or to any global objective. The convergence emerges from value concentration dynamics: small-valued cells persist while large-valued cells disproportionately accumulate mass, reducing the sum of reciprocals over time.

While the system does not converge exactly to $e$ in all runs, the robustness of this attractor across diverse initial conditions suggests a nontrivial relationship between local multiplicative dynamics and exponential scaling laws.

### *Locality as a Beneficial Constraint*

Inspection distance had a strong and monotonic effect on convergence quality (Figure 2). Immediate neighborhood inspection ($d = 1$) produced the closest approach to $e$ while increasing $d$ led to progressively worse performance.

| **Distance** | **Mean** $S$ | $\lvert S - e \rvert$ |
|---|---|---|
| $d = 1$ | 2.557 | **0.161** |
| $d = 2$ | 2.253 | 0.465 |
| $d = 3$ | 2.047 | 0.671 |
| $d = 4$ | 1.834 | 0.884 |
| $d = 5$ | 1.641 | 1.077 |

Larger inspection ranges accelerate value concentration by enabling cells to identify globally maximal targets. However, this acceleration induces overshoot: the system passes through the attractor region and asymptotically continues towards $S = 1$. Locality thus functions as a stabilizing constraint, preventing excessive cascade amplification.

This mirrors observations in developmental biology, where short-range signaling often yields more reliable pattern formation than long-range coordination (Fields & Levin, 2022).

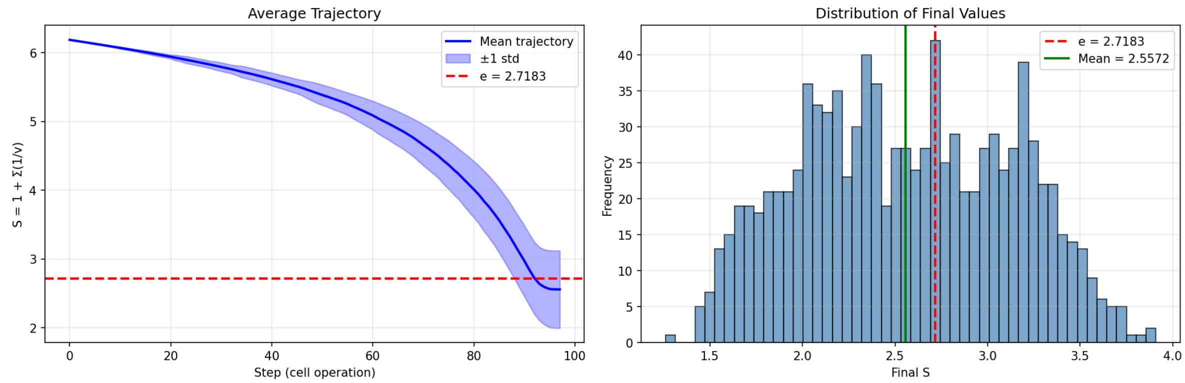


Figure 1: Emergent Convergence Trajectory. Left: Step-by-step trajectory of $S$ during a single round, showing descent from initial values toward $e$ (dashed line). The system naturally approaches Euler's number without any explicit encoding of this target. Right: Distribution of final $S$ values across 1,000 iterations, centered near $e$ with characteristic variance reflecting sensitivity to initial conditions and execution order.

### ***Barriers Induce Enhanced Competency***

The central finding of this work concerns the effect of barriers (Figure 3). Using optimal distance $d = 1$:

| Type | Mean $S$ | $\|S - e\|$ | Improvement |
|---|---|---|---|
| none | 2.557 | 0.161 | baseline |
| h_wall | 2.557 | 0.161 | $-0.2\%$ |
| v_wall | 2.576 | 0.143 | $+11\%$ |
| cross | 2.592 | 0.126 | $+22\%$ |
| **double_h** | **2.611** | **0.107** | **+34%** |
| box | 2.540 | 0.178 | $-10\%$ |

The double horizontal wall configuration type - walls at rows 3 and 6 dividing the grid into three horizontal bands - achieves the closest approach to $e$, improving convergence by 34%. The cross pattern, dividing the grid into four semi-quadrants, improves by 22%.

Critically, the small 2×2 enclosed box *worsens* convergence by 10%. This demonstrates that barrier placement should not isolate cellular areas as isolation promotes blind co-evolution and does not induce systemic improvement towards a latent goal. Effective constraint must operate at the organizational level of the entire collective.

### ***Mechanistic Analysis: Two Pathways to Agentic Expression***

Propagation analysis reveals *how* barriers improve convergence (Figure 4). We tracked every multiplication event, computing "generation numbers" for each cell:

- **Generation 0**: Cells that acted but were never targeted (pure initiators)
- **Generation $N$**: Cells first targeted by a Generation $N-1$ cell
- **Non-participants**: Cells involved in no multiplications

Generational analysis was performed for the previously two best performing wall configuration types: cross and double horizontal (double_h). These are compared to a no wall configuration (none). Averages were determined across 1,000 iterations to determine the average maximum generation (AMG), average non-participants (ANP), and average final $S$ (AF$S$).

| Type | AMG | ANP | AF$S$ |
|---|---|---|---|
| none | 19.6 | 34.7% | 2.56 |
| double_h | 15.9 | 39.2% | 2.61 |
| cross | 16.5 | 38.7% | 2.59 |

Two distinct observations emerge:

*Isolation:* Average non-participants increase from 34.7% to 39.2% for the double_h configuration type. Cells at wall boundaries have their inspection distance impeded, reducing total multiplication activity. The system achieves better convergence through decreased participation.

*Cascade Limitation:* Average maximum generation depth drops dramatically from 19.6 to 16.5 in the

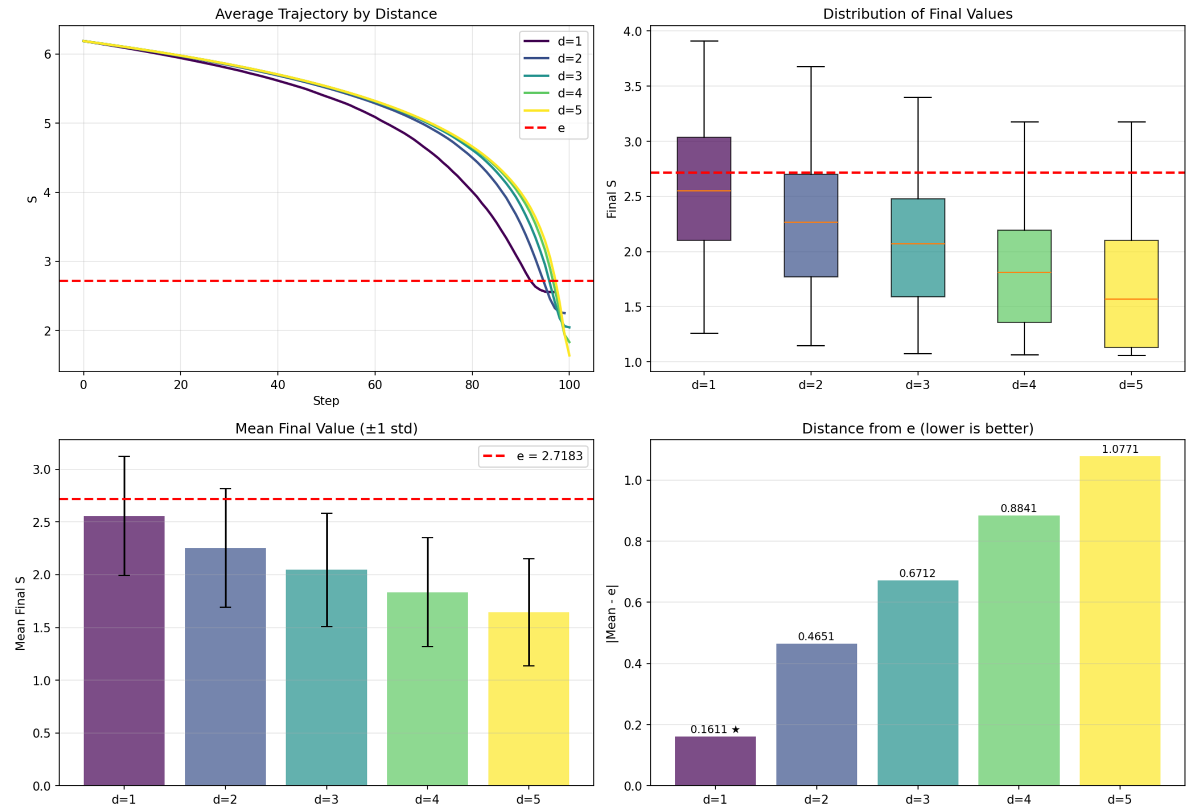


Figure 2: Inspection Distance Optimization. Top Left: Average trajectories showing increasing overshoot related to larger inspection distances 1–5. Top Right: Box plots showing final $S$ distributions for inspection distances 1–5. Distance $d = 1$ centers closest to $e$ (dashed red line), with increasing overshoot as distance grows. Bottom Left: Mean distance from $e$ demonstrates monotonic degradation with increased inspection range. Bottom Right: Absolute distance from $e$. Locality is essential for optimal convergence - extended reach causes the system to overshoot its natural attractor.

cross configuration type. Further to 15.9 in the double_h configuration type. This constrains generational propagation depth in participating cells.

## Discussion

### *Latent Competency in Minimal Systems*

Our cellular automaton exhibits what we term *latent competency*: the capacity to reliably approach a mathematically significant target state through purely local interactions. No cell “knows” about $e$; no global controller guides the process. Yet the collective consistently converges toward this fundamental constant (Kauffman, 1993).

This aligns with the framework proposed by Zhang et al. (2024), who demonstrated that emergent capacities in simple systems enlightens a new perspective to the field of Diverse Intelligence, showing how basal forms of latent agentic competency can emerge without being explicitly encoded. Supporting the view that latent capacities can emerge in minimal systems when familiar rules are instantiated as distributed, embodied processes rather than abstract procedures (Levin, 2023b).

The key insight is that intelligence - understood as competency in navigating toward goals despite obstacles (James, 1890) - need not require complex substrates. Simple local rules, operating in parallel across a distributed system, can produce goal-directed collective behavior that exceeds what any individual component could achieve (Reynolds, 1987).

### *Constraint as a Competency-Inducing Factor*

Contrary to intuition, obstacles improved performance (Crosscombe & Lawry, 2023). This is not

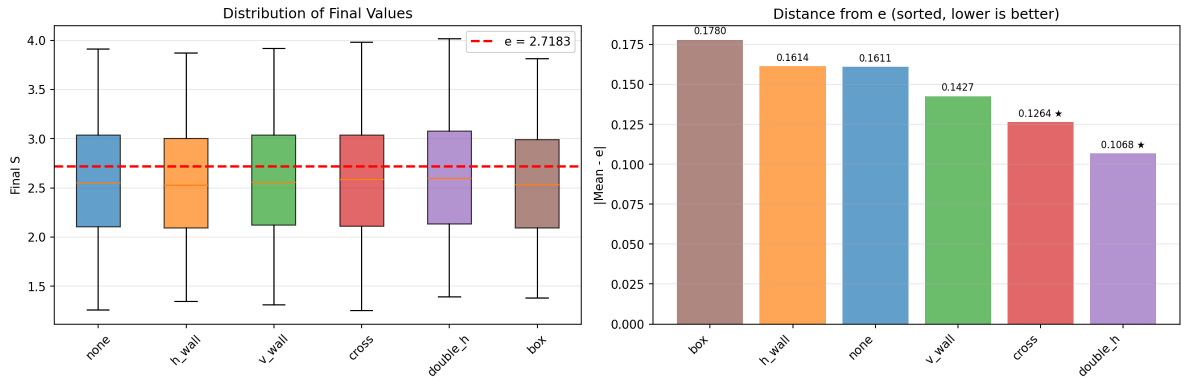


Figure 3: Wall Configuration Analysis. Left: Distribution of final $S$ values for each wall configuration. Double horizontal walls (double_h) produce the distribution most tightly centered around $e$. Right: Distance from $e$ sorted from greatest to least. Double horizontal walls achieve the minimum distance (0.107), a 34% improvement over baseline. Stars indicate the two best-performing configurations.

because walls encode information about the goal, but because they constrain runaway cascades that otherwise destabilize the dynamics.

A single multiplication can trigger a chain reaction up to 23 generations deep. Each generation amplifies the concentration effect, and the cumulative result overshoots the natural attractor $e$.

By truncating interaction pathways, barriers force the system to rely on bounded, local processes. The result is not slower convergence, but more precise convergence—a refinement rather than an impairment of collective behavior (Bak et al., 1987).

Walls interrupt these runaway cascades. By forcing the collective to achieve its goal through more constrained pathways, barriers induce a more *refined* form of collective behavior. The system cannot rely on long-range cascade effects; it must reach $e$ through local, bounded interactions.

This principle - that appropriate constraint induces enhanced competency - has broad implications:

*Biological development:* Tissue boundaries and physical constraints help establish correct anatomical patterns. Zhang et al. (2024) notes that ”morphogenesis is the behavior of a collective intelligence of cells in anatomical morphospace,” and self-imposed system constraints may play a crucial role in constraining this collective toward correct outcomes.

*Distributed computing:* Network partitioning, often seen as a failure mode, may in some contexts improve collective computation.

*Organizational design:* Appropriate boundaries between teams or departments may induce more refined problem-solving than fully connected structures that permit cascade effects.

### ***Scale-Appropriate Barriers***

The failure of the 2×2 box to improve, and instead hurting, convergence reveals an important constraint: barriers must still allow for organizational communication over complete isolation. The box creates an isolation that causes separate evolution of the inner and outer locations of the boxed wall.

In contrast, the double horizontal walls divide the *entire* grid into semi-independent bands. The cross pattern creates four quadrants. These configurations operate at the scale of the collective itself, fundamentally restructuring how cascades can propagate.

This scale-sensitivity suggests a design principle: to induce competency through constraint, barriers must match the organizational scale of the target behavior. Local obstacles produce local perturbations; systemic improvement requires systemic constraint.

### ***The Emergence of e***

The emergence of $e$ reflects the system's underlying multiplicative structure. The dynamics resemble stochastic, discrete analogs of exponential growth and decay processes, long associated with

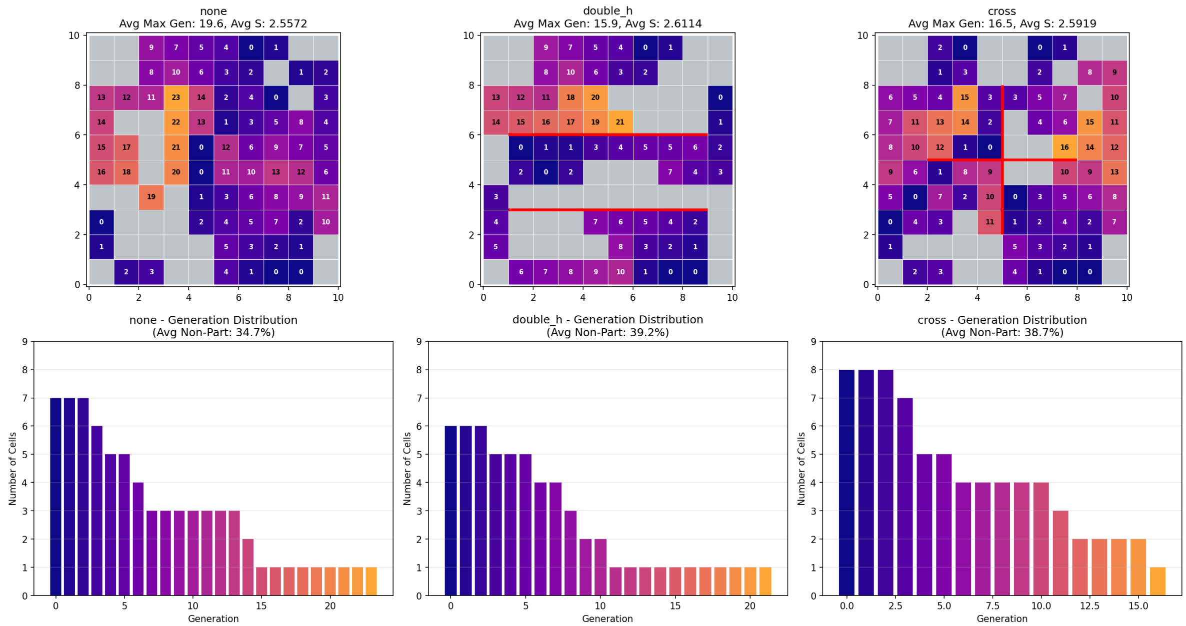


Figure 4: Propagation Analysis; Random Example Heat Maps and Generation Depths. Top row: Generation heat maps showing when each cell was first multiplied. Darker colors indicate earlier generations (closer to initiation); lighter colors indicate later generations (deeper in the cascade). The cross pattern shows dramatically shallower maximum depth (16 vs 23). Walls appear as red lines. Bottom row: Generation distributions for the random heat mapped examples.

$e$ through limits and series expansions.

The identity $e = \lim_{n\to\infty}(1 + 1/n)^n$ and the series $e = \sum 1/n!$ both involve multiplicative accumulation. Our cellular automaton are implementing a discrete, stochastic analog of these processes.

A formal derivation of these effects remains an open problem. However, the empirical emergence of the attractor suggests that Euler's number arises naturally in distributed systems governed by local multiplicative accumulation and bounded propagation (Mitchell, 2009).

## Conclusion

A cellular automaton governed by a simple local multiplication rule exhibits robust convergence toward Euler's number. Introducing appropriately scaled physical barriers improves this convergence by constraining cascade depth and interactions.

Key findings:

1. **Emergent mathematical convergence:** The system approaches $e$ when properly constrained without any explicit encoding of this target.
2. **Locality is essential:** Inspection distance $d = 1$ produces optimal results; delocalization causes overshoot.
3. **Barriers induce competency:** Double horizontal walls improve convergence by 34% by constraining cascade propagation.
4. **Two observations:** Isolation (increasing non-participants) and cascade limitation (reducing generation depth) both improve convergence.

These results contribute to the emerging understanding that latent basal competencies - goal-directed behaviors not explicitly programmed - can arise in minimal systems through collective dynamics. The principle that appropriate constraint induces enhanced competency offers a new lens for understanding distributed systems in biology, computation, and beyond.

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